

# **Research at the K-12 Level: Considerations and Challenges**

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# Research on K-12 Mathematics Teaching and Learning: Three Central Themes

- **The goal of mathematics education is not simply for students to learn to “do math,” but for all students to understand the mathematics they do.**
- **All students can understand mathematics. They differ in how quickly and easily they learn.**
- **Students’ meaningful learning of mathematics involves not only what they are taught, but also how they are taught.**

# What does K-12 Mathematics Entail?

## ➤ School mathematics has changed over time.

- ❑ 30 years ago most states only required one course in high school math for graduation, and general math would suffice.

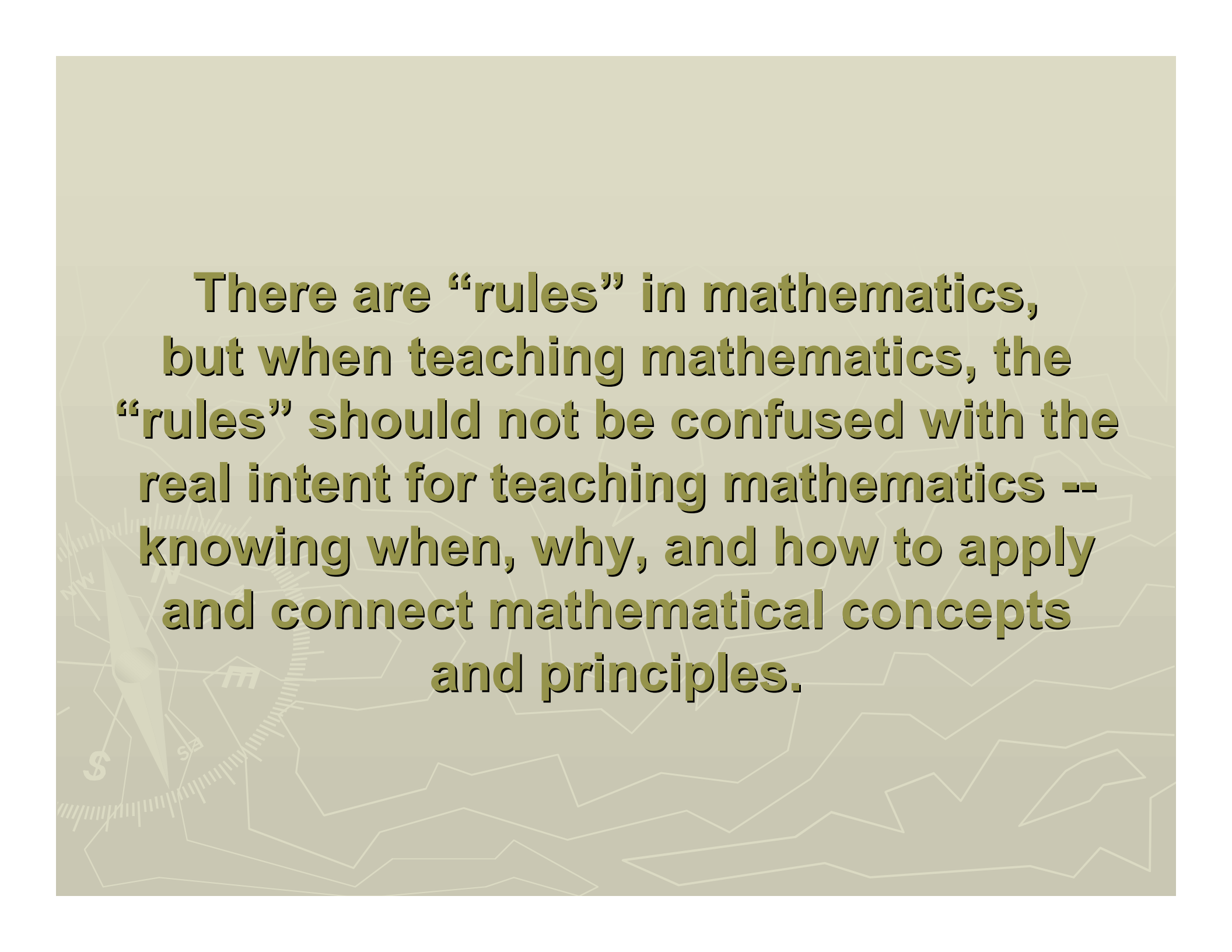
- ❑ In Maryland now, 3 courses are required including “topics from” algebra, data analysis and geometry.

- ❑ 30 years ago, K-8 math was primarily computation.

- ❑ Now NAEP measures number and operations, plus geometry, measurement, data analysis, probability, and algebra.

**Nobel Laureate Richard Feynman's comparison of the principles of physics to chess, also apply to mathematics teaching and learning.**

**The rules of chess are easy to state, but knowing the rules of chess is not the same as playing chess, as following the plays of a game of chess, or as predicting what will happen next in a chess game.**

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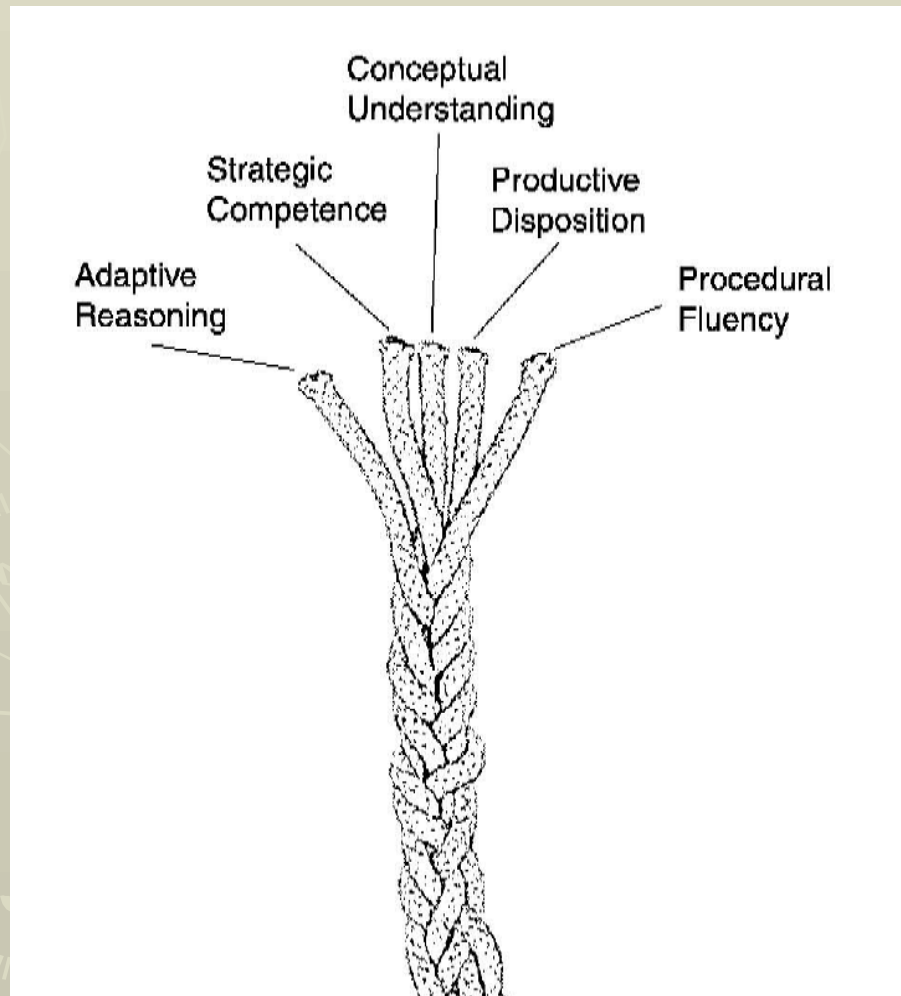
**There are “rules” in mathematics,  
but when teaching mathematics, the  
“rules” should not be confused with the  
real intent for teaching mathematics --  
knowing when, why, and how to apply  
and connect mathematical concepts  
and principles.**

## Two References for the Information that Follows

**Donovan, M. S., & Bransford, J. D. (Eds.) (2005). *How students learn: Mathematics in the classroom*. Washington, DC: National Academy Press.**

**Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.**

# Mathematical Proficiency



- **Conceptual Understanding --** Comprehension of mathematical concepts, operations, and relations
- **Procedural Fluency --** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic Competence --** Ability to formulate, represent, and solve mathematical problems.
- **Adaptive Reasoning --** Capacity for logical thought, reflection, explanation, and justification
- **Productive Disposition --** Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

# Learning Mathematics

- **Students have prior knowledge that they bring to school.**
  - ❑ **If that knowledge is not accessed during instruction, students may have difficulty or fail to learn.**
  - ❑ **If that knowledge is not accessed during instruction, students may learn new content for a short time, but then forget it -- slipping back only to their prior knowledge.**

# Learning Mathematics

➤ **Students who learn and retain what they have learned actually understand.**

❑ **It's not just a matter of retaining factual knowledge and skills.**

❑ **What is required is to grasp the concepts and ideas underlying those facts and skills.**

❑ **What is required is to mentally organize all those facts, skills, concepts and ideas in a way that makes sense *to the learner*.**

❑ **This organization supports future recall, access, and use.**

# Learning Mathematics

- **Successful learners consciously “take charge” and monitor their own learning.**
  - ❑ **They reflect on what they do and do not understand (and ask for help or work to make sense).**
  - ❑ **They use strategies that they know help them to learn.**
    - ✓ **They ask questions, explain to themselves and others, and recheck what they have done, read and written.**
    - ✓ **In mathematics, they ask if their answer “makes sense” and carry out steps to determine that.**

# Learning Mathematics

- **Distributed practice is important for skill learning.**
  - ❑ Practice permits the steps making up a skill to become connected and stored as a single procedure in one's memory.
  - ❑ This supports recall and automatic execution of the skill.
- **Conceptual understanding also supports recall and use of skills.**
  - ❑ It is easier to use an understood procedure.
  - ❑ If a skill or procedure is understood, it is easier to realize to use it when faced with a new situation or to realize that all that is needed is an adjustment in order to use it to solve a new problem.

# Learning Mathematics

- **Students who understand why and how a procedure works will develop accurate recall faster, with less practice.**
- **If students practice what they do not understand, it is hard for them to go back and learn underlying meaning.**
- **Practice without understanding can lead to consistent procedural errors that are hard to eliminate.**

# Teaching Mathematics

- **Build on students' current knowledge and skills.**
  - ❑ **Not just a pretest to target review and re-teaching. This can actually build a memorize-forget cycle.**
  - ❑ **Access students' thinking. Ask questions that require students to explain their ideas and strategies, even if they are inaccurate or incomplete.**

# Teaching Mathematics

- **Balance discussion of student ideas with consideration of intended new mathematical content.**



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**Show and tell**

**Demonstrate and practice**

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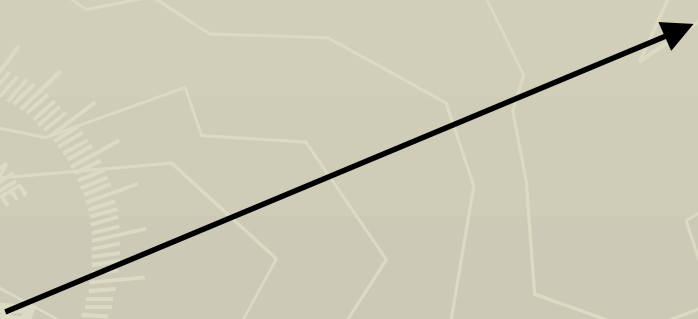
- **Balance discussion of student ideas with consideration of intended new mathematical content.**

**Asking questions**

**Questioning answers**

**Show and tell**

**Demonstrate and practice**



**Theorem: Never tell a student what that student could tell you (if you asked).**

**Corollary: Never tell a student what another student could tell that student for you.**

# Teaching Mathematics

- **Integrate concepts and skills during instruction.**
  - ❑ **Question students to explain how they solved a problem and how they knew to do what they did (or why they did what they did).**
  - ❑ **Engage the class in considering differing methods, in identifying advantages or disadvantages of methods, in determining if a method is mathematically correct and rigorous, in identifying points of confusion, and in asking questions.**

# Teaching Mathematics

- **Mathematics is part invention and part convention.**
  - ❑ **Terminology and notation require more explicit presentation.**
  - ❑ **Standards for mathematical correctness, generalizability, and efficiency still hold.**
  - ❑ **Consolidating expressed ideas and phrasing subsequent principles generally require more “teacher talk.”**
- **Teachers must decide when to ask, when to listen, and when to tell.**

# Teaching Mathematics

- Too often what students learn from monitoring their own learning in mathematics classes is that they are “not good at math” and that math is confusing and difficult.
- Instruction that expects students to ask questions and to explain their reasoning pushes students to:
  - ❑ reflect on their understanding,
  - ❑ organize their thinking,
  - ❑ self-monitor and
  - ❑ persist.

# Other School-based Influences

- **Complete and coherent mathematics curriculum that portrays a connected and increasing mathematical trajectory.**
  - ❑ **Conveys pacing, order, emphasis and expectations**
  - ❑ **References available mathematical materials**
  - ❑ **Aligned with standardized assessments**
- **Students learn what they have an opportunity to learn (Hiebert, 2003).**
  - ❑ **Most current curricula and state standards focus more on calculating, labeling, defining, and recalling in problem exercises than on reasoning, communicating, connecting, justifying, conjecturing and problem solving.**

# Mathematics Content

Too often the “conventional wisdom” is that the curriculum in high poverty classrooms should follow a fixed sequence of lessons, emphasizing practice and moving from basic to more advanced skills without ever addressing problem solving or reasoning.

This is not supported by research, if your intent is to advance mathematical learning and comprehension.

# **Theoretical Goal**

**Mathematics in the  
State framework**

**Mathematics in the  
State Test**

**Knowledgeable teachers using new  
instructional strategies and resources to  
foster student learning of a legitimate  
trajectory of rich mathematics**

**Coherent  
Curriculum**

**Textbooks**

**Assessments**



# Professional Development

- **Sustained professional development can positively impact both instructional practice and student achievement.**
- **It is the key to local implementation.**
- **Must integrate mathematics content and pedagogy, target the curriculum, access only available resources, address teaching demands of the teachers, and be consistently attended.**
- **Requires sustained administrative resolve.**

# What Research Cannot Tell Us

- **Standards are not determined by research.**
- **Standards are statements of priorities and goals.**
- **Standards imply beliefs about mathematics and mathematics learning and teaching.**
- **What is always “best” cannot be determined by research.**
- **Research cannot imagine new ideas.**

# What Research Can Tell Us

- **Can document the current situation.**
- **Can document the effectiveness of a new idea.**
- **Can suggest explanations for success or failure.**

# Thinking about Working with a K-12 District? A University Researcher's Perspective

- **First, establish a shared commitment.**
  - ❑ **Learn of and then recognize the unique strengths that each group brings to the enterprise.**
  - ❑ **Establish a common ground built on shared interests and goals.**
  - ❑ **Work together on “first steps.”**
  - ❑ **Communicate, communicate, communicate.**

The background features a light beige color with a faint, stylized line graph and a compass rose. The compass rose is located on the left side, showing cardinal directions (N, S, E, W) and a dollar sign (\$) near the bottom. The line graph consists of several jagged, interconnected lines that suggest a complex data set or a path.

**Recognize that Collaboration  
is NOT Enough.**

**Meeting Accepted Obligations  
is NOT Enough.**

➤ **Second, work towards mutual obligation.**

- Support and learn from each other.**
- Recognize and address needs of each other.**
- Recognize and accept the strengths of each other.**
- Interact to advance the work of the partnership.**

# **You Cannot Control the Wind, but You Can Adjust Your Sail**

- **Establish and maintain a connection with district's Research, Evaluation and Assessment Division, although this leadership will be transient.**
- **Constantly examine the alignment and connections between curriculum, instructional materials, assessment, and professional development.**
- **Set up a system to collect teacher/school data to measure the fidelity of the implementation.**

- **Earn the acceptance, trust and respect of the school system personnel through immersion in the day-to-day reality of the schools.**
- **Carve time to reflect on the effort.**
- **Be prepared to withstand political and administrative upheaval.**
- **Be prepared to positively react to the crisis of the moment and to fill many voids as the resident expert.**
- **Know what the system will bear in terms of reform and funding.**

- **Remember, the goal is to stimulate and study credible efforts -- not just efforts that align with theory.**
- **Realize that the demands of reform will interfere with university expectations.**
- **If you crave recognition, don't do this. The university partner generally is not acknowledged in public venues (unless the university is the speaker).**

# Lessons Learned

- **Encourage central administration resolve.**
- **Engage administrators at all levels in understanding what is involved. (When consistent valued results are presumed, support declines.)**

- **Work with district to establish mechanisms for supporting and monitoring implementation without attempting to control or define details of individual classroom instruction.**
- **Tell the story and write before accepting “the next” project.**
- **Let your university or college know your needs; ask for help when needed. (Take advantage of your advantage.)**

# Parting Thoughts

- **Increased student achievement in mathematics occurs when there is a positive change in the reality of instructional practice experienced by students and teachers within and across schools.**
- **Capacity for reform occurs when knowledgeable teachers and administrators, with sufficient resources, are committed to the premise that change is important -- not simply because of a mandate for improvement, but because the intended practices will permit teachers and students to engage in more exciting, challenging and productive instruction.**